

## 4.1 High-Frequency Transistor Models: Capacitive Effects in BJTs and FETs

At low and mid-frequencies, transistor behavior is primarily characterized by resistive and transconductance elements in their small-signal models. However, as the operating frequency of the signal increases, the inherent parasitic capacitances within the physical structure of these transistors can no longer be ignored. These internal capacitances provide low-impedance paths for the signal at high frequencies, effectively diverting current away from the amplifying terminals and leading to a significant reduction in amplifier gain. Understanding these capacitive effects is paramount for accurately predicting and designing high-frequency amplifier circuits.

### 4.1.1 Capacitive Effects in BJTs (Bipolar Junction Transistors)

BJTs, due to their P-N junction construction, possess several internal capacitances that profoundly impact their high-frequency performance. These capacitances can be broadly categorized as junction capacitances and diffusion capacitance.

- **Junction Capacitances (Depletion Region Capacitances):** These arise from the charge storage effects in the reverse-biased depletion regions of the P-N junctions. As the voltage across a reverse-biased junction changes, the width of the depletion region changes, leading to a capacitance effect.
  - **Collector-Base Junction Capacitance ( $C_{\mu}$  or  $C_{cb}$ ):** This capacitance exists across the reverse-biased collector-base junction. In the hybrid-pi model, it is denoted as  $C_{\mu}$ . This capacitance is particularly critical because it connects the input (base) to the output (collector) of the transistor. Due to the Miller Effect (discussed below),  $C_{\mu}$  can be effectively multiplied at the input of common-emitter amplifiers, drastically increasing the input impedance seen by the signal source and significantly limiting the amplifier's upper frequency response. This is often the dominant factor in determining the high-frequency cutoff for common-emitter configurations.
  - **Emitter-Base Junction Capacitance ( $C_{je}$ ):** This capacitance exists across the forward-biased emitter-base junction. While it has a depletion component, its dominant part in forward bias is the diffusion capacitance.
- **Diffusion Capacitance ( $C_d$  or  $C_{\pi}$ ):** This capacitance is associated with the storage of minority charge carriers (electrons injected into the p-type base from the emitter, and holes injected into the n-type emitter from the base) in the neutral regions of a forward-biased P-N junction. When the emitter-base junction is forward-biased and conducting, a significant amount of charge is injected and stored in the base region. If the input signal frequency changes rapidly, this stored charge needs to be quickly added or removed, which takes time and presents a capacitive impedance. The  $C_{\pi}$  parameter in the hybrid-pi model primarily represents this diffusion capacitance along with the smaller depletion capacitance of the emitter-base junction. Its value is directly proportional to the DC emitter current (and thus  $I_c$ ) and inversely proportional to the transistor's cut-off frequency ( $f_T$ ), reflecting the speed at which the transistor can respond to changes in input current.

**High-Frequency Hybrid-Pi Model for BJT:** To analyze the high-frequency behavior of a BJT, the small-signal hybrid-pi model is augmented with these parasitic capacitances. The key components include:

- $r_{\pi}$ : The input resistance from base to emitter, representing the dynamic resistance of the forward-biased base-emitter junction. It is calculated as  $r_{\pi} = \beta / g_m$ , where  $\beta$  is the common-emitter current gain and  $g_m$  is the transconductance.
- $g_m$ : The transconductance, which relates the change in collector current to the change in base-emitter voltage ( $g_m = I_c / V_T$ , where  $V_T$  is the thermal voltage).
- $r_o$ : The output resistance from collector to emitter, representing the Early effect.
- $C_{\pi}$ : The total capacitance between base and emitter, primarily diffusion capacitance.
- $C_{\mu}$ : The capacitance between collector and base, primarily junction capacitance.

At high frequencies, the reactances of  $C_{\pi}$  and  $C_{\mu}$  become small enough to shunt current away from  $r_{\pi}$  and the collector, respectively, leading to a decrease in gain.

#### 4.1.2 Capacitive Effects in FETs (Field-Effect Transistors)

FETs (JFETs and MOSFETs) also exhibit internal capacitances that limit their high-frequency performance. These are primarily derived from the gate electrode's proximity to the channel and source/drain regions.

- **Gate-Source Capacitance ( $C_{gs}$ ):** This capacitance exists between the gate and the source terminals. In MOSFETs, it is primarily due to the gate oxide acting as a dielectric separating the gate electrode from the channel and source region. In JFETs, it is the junction capacitance of the reverse-biased gate-source junction. It is generally the largest of the FET capacitances.
- **Gate-Drain Capacitance ( $C_{gd}$ ):** This capacitance exists between the gate and the drain terminals. Similar to  $C_{\mu}$  in BJTs, this capacitance is subject to the Miller Effect and is often the most significant factor limiting the bandwidth of common-source FET amplifiers. It acts as a feedback path from output to input.
- **Drain-Source Capacitance ( $C_{ds}$ ):** This capacitance exists between the drain and source terminals. It is generally smaller than  $C_{gs}$  and  $C_{gd}$  and often has less impact on the overall frequency response unless other capacitances are extremely small or the load is very high impedance.

**High-Frequency FET Model:** The high-frequency small-signal model for FETs includes these capacitances:

- $g_m$ : The transconductance, relating the change in drain current to the change in gate-source voltage.
- $r_o$ : The output resistance from drain to source, representing the channel length modulation effect.
- $C_{gs}$ : Capacitance between gate and source.
- $C_{gd}$ : Capacitance between gate and drain.
- $C_{ds}$ : Capacitance between drain and source.

**Miller Effect (Detailed):** The Miller effect is a phenomenon that significantly impacts the input capacitance of an inverting voltage amplifier. When a capacitance (like  $C_{\mu}$  in a BJT or

Cgd in an FET) is connected between the input and output terminals of an amplifier with voltage gain ( $A_v$ ), it appears at the input side as an effectively much larger capacitance.

- **Mechanism:** Consider  $C_\mu$  connected between the base and collector of a common-emitter amplifier. If the input voltage changes by  $\Delta V_{in}$ , the output voltage changes by  $A_v * \Delta V_{in}$ . Since  $A_v$  is negative for an inverting amplifier, the voltage change across  $C_\mu$  is  $\Delta V_{C_\mu} = \Delta V_{in} - \Delta V_{out} = \Delta V_{in} - (A_v * \Delta V_{in}) = \Delta V_{in} (1 - A_v) = \Delta V_{in} (1 + |A_v|)$ .
  - The current flowing through  $C_\mu$  is  $I_{C_\mu} = C_\mu * d(V_{C_\mu})/dt$ .
  - The effective input capacitance at the base is  $C_{in(Miller)} = I_{C_\mu} / (d(\Delta V_{in})/dt) = C_\mu * (1 + |A_v|)$ .
  - **Impact:** This amplified input capacitance ( $C_{in(Miller)}$ ) interacts with the source resistance ( $R_s$ ) connected to the input of the amplifier, forming an RC low-pass filter with a time constant  $\tau = R_s * C_{in(Miller)}$ . This filter then dictates the upper cutoff frequency ( $f_H$ ) of the amplifier:  $f_H = 1 / (2\pi * R_s * C_{in(Miller)})$ . A larger  $C_{in(Miller)}$  leads to a lower  $f_H$ , thus limiting the amplifier's bandwidth. The Miller effect is often the dominant factor causing high-frequency roll-off in common-emitter and common-source amplifiers.
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## 4.2 Frequency Response of Single-Stage Amplifiers

The frequency response of an amplifier describes how its gain (and phase shift) varies as a function of the input signal frequency. An ideal amplifier would maintain a constant gain across all frequencies, but practical amplifiers always have a limited range of frequencies over which they provide useful amplification. This operating range is defined by specific cutoff frequencies.

**4.2.1 Gain-Frequency Plot:** The frequency response is typically represented graphically by a Bode plot. This plot usually shows the magnitude of the voltage gain (in decibels, dB) on the y-axis against frequency (on a logarithmic scale) on the x-axis. A flat region in the middle represents the "mid-band," where the gain is relatively constant.

**4.2.2 Upper and Lower Cutoff Frequencies:** An amplifier's effective frequency range, known as its bandwidth, is bounded by two critical frequencies, also called the -3 dB frequencies or half-power frequencies. At these frequencies, the power delivered to the load is half the maximum mid-band power, or equivalently, the voltage gain drops to 0.707 (or  $1/\sqrt{2}$ ) times its mid-band gain. In decibels, 0.707 corresponds to a -3 dB drop from the mid-band gain.

### 1. Lower Cutoff Frequency ( $f_L$ or $f_1$ ):

- **Definition:** The frequency at which the amplifier's voltage gain drops to 0.707 of its mid-band gain as frequency decreases.
- **Cause:** Primarily determined by the large external capacitors used for coupling and bypassing.
  - **Coupling Capacitors:** Input coupling capacitors (between signal source and amplifier input) and output coupling capacitors (between amplifier output and load) are used to block DC components while

allowing AC signals to pass. At very low frequencies, the reactance of these capacitors ( $X_c = 1 / (2\pi fC)$ ) becomes very high, effectively forming a high-pass filter. This high reactance significantly impedes the AC signal flow, leading to a reduction in input signal reaching the amplifier or output signal reaching the load, thus reducing gain.

- **Bypass Capacitors:** Bypass capacitors (e.g., across the emitter resistor in a BJT common-emitter amplifier, or source resistor in an FET common-source amplifier) are used to provide an AC ground path, preventing AC voltage drops across these resistors which would otherwise introduce negative feedback and reduce gain. At low frequencies, the bypass capacitor's reactance increases, reducing its bypassing effectiveness. This reintroduces degenerative feedback, causing the gain to drop.
- **Formula (for a single dominant RC high-pass network):** The lower cutoff frequency associated with a specific capacitor and its Thévenin equivalent resistance (seen by the capacitor) is given by:  $f_L = 1 / (2\pi * R_{Th} * C)$  Where  $R_{Th}$  is the equivalent resistance seen by the capacitor. For a complete amplifier, the overall  $f_L$  is influenced by all such low-frequency RC networks. The highest of these individual  $f_L$  values often dictates the overall amplifier's lower cutoff frequency.

## 2. Upper Cutoff Frequency ( $f_H$ or $f_2$ ):

- **Definition:** The frequency at which the amplifier's voltage gain drops to 0.707 of its mid-band gain as frequency increases.
- **Cause:** Primarily determined by the internal parasitic capacitances of the transistor ( $C_\pi$ ,  $C_\mu$  for BJT;  $C_{gs}$ ,  $C_{gd}$ ,  $C_{ds}$  for FET) and any stray wiring capacitances.
  - At high frequencies, the reactances of these small capacitances become low enough to provide alternative, low-impedance paths.
  - For example,  $C_\pi$  and the Miller-effect amplified  $C_\mu$  (or  $C_{gd}$ ) shunt the input signal current, effectively reducing the signal reaching the transistor's active region.
  - Similarly, internal capacitances at the output (e.g.,  $C_{ds}$ ,  $C_\mu$ ) can shunt the output signal.
  - These parasitic capacitances form low-pass filter networks with the effective resistances at various nodes, causing the gain to roll off.
- **Formula (for a single dominant RC low-pass network):** The upper cutoff frequency associated with a specific capacitance and its Thévenin equivalent resistance is given by:  $f_H = 1 / (2\pi * R_{Th} * C)$  Where  $R_{Th}$  is the equivalent resistance seen by the capacitor. For a complete amplifier, the overall  $f_H$  is influenced by all such high-frequency RC networks. The lowest of these individual  $f_H$  values often dictates the overall amplifier's upper cutoff frequency.

**4.2.3 Mid-Band Gain ( $A_{v\_mid}$ ):** This is the maximum and relatively constant voltage gain of the amplifier in the frequency range between  $f_L$  and  $f_H$ . In this mid-band region, the reactances of the coupling and bypass capacitors are effectively zero (acting as short circuits), and the reactances of the internal parasitic capacitances are effectively infinite (acting as open circuits). Therefore, the frequency-dependent components have negligible

effect, and the amplifier's gain is determined solely by its resistive network and transistor parameters ( $g_m$ ,  $r_{\pi}$ ,  $r_o$ ).

**4.2.4 Bandwidth (BW):** The bandwidth of an amplifier is the range of frequencies over which the amplifier provides useful amplification, typically defined as the range between the lower and upper cutoff frequencies. **Formula:**  $BW = f_H - f_L$  For most broadband amplifiers,  $f_L$  is significantly smaller than  $f_H$  (e.g.,  $f_L$  in Hz,  $f_H$  in MHz), so the bandwidth is often approximated as  $BW \approx f_H$ .

**Numerical Example 4.2.4:** A single-stage common-emitter amplifier has a mid-band voltage gain of 80. Its input coupling capacitor, along with the input resistance, results in a lower cutoff frequency of 30 Hz. The combined effect of the transistor's internal capacitances and the Miller effect capacitance at the input results in an upper cutoff frequency of 750 kHz.

- **Problem:** Calculate the amplifier's bandwidth and the gain at  $f_H$  in dB relative to the mid-band gain.
  - **Given:**  $f_L = 30$  Hz,  $f_H = 750$  kHz,  $A_{v\_mid} = 80$ .
  - **Part A: Calculate Bandwidth.**
    - **Formula:**  $BW = f_H - f_L$
    - **Calculation:**  $BW = 750,000 \text{ Hz} - 30 \text{ Hz} = 749,970 \text{ Hz}$
    - **Result:** The bandwidth of the amplifier is approximately 749.97 kHz.
  - **Part B: Calculate Gain at  $f_H$  in dB relative to mid-band.**
    - **Principle:** By definition, at the cutoff frequencies ( $f_L$  and  $f_H$ ), the voltage gain drops to 0.707 times the mid-band gain.
    - **Gain at  $f_H$  (linear scale):**  $A_{v\_at\_fH} = 0.707 * A_{v\_mid} = 0.707 * 80 = 56.56$ .
    - **Mid-band Gain in dB:**  $A_{v\_mid\_dB} = 20 * \log_{10}(A_{v\_mid}) = 20 * \log_{10}(80) = 20 * 1.903 = 38.06 \text{ dB}$ .
    - **Gain at  $f_H$  in dB:**  $A_{v\_at\_fH\_dB} = 20 * \log_{10}(A_{v\_at\_fH}) = 20 * \log_{10}(56.56) = 20 * 1.752 = 35.04 \text{ dB}$ .
    - **Difference:**  $A_{v\_at\_fH\_dB} - A_{v\_mid\_dB} = 35.04 \text{ dB} - 38.06 \text{ dB} = -3.02 \text{ dB}$ .
    - **Result:** As expected from the definition, the gain at  $f_H$  is approximately -3 dB lower than the mid-band gain.
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## 4.3 Frequency Response of Multistage Amplifiers

When two or more amplifier stages are cascaded (connected in series, where the output of one stage feeds the input of the next) to achieve a higher overall gain, their individual frequency responses interact to determine the overall frequency response of the entire system. Understanding this interaction is crucial because the overall bandwidth is generally *not* simply the sum or average of individual stage bandwidths.

### 4.3.1 Overall Bandwidth Considerations:

The overall bandwidth of a cascaded amplifier is generally *less* than the bandwidth of any single stage. This reduction occurs because each amplifier stage acts as a frequency filter. When multiple filters are cascaded, their filtering effects compound. Frequencies that are

already attenuated by one stage will be further attenuated by subsequent stages, leading to a sharper overall roll-off and a narrower effective bandwidth.

- **Overall Lower Cutoff Frequency ( $f_L(\text{overall})$ ):**

- The overall lower cutoff frequency of a cascaded amplifier is primarily determined by the *highest* of the individual lower cutoff frequencies of the stages. If one stage has an  $f_L$  of 100 Hz and another has 50 Hz, the 100 Hz cutoff will dominate the low-frequency performance of the entire amplifier, as signals below 100 Hz will be significantly attenuated by that particular stage.
- For  $N$  identical cascaded stages, the overall lower cutoff frequency is slightly higher than that of a single stage. The precise calculation involves complex formulas. A commonly used approximation (though not always exact, especially for non-identical stages) for  $N$  identical stages where  $f_L$  is due to a single dominant pole is:  $f_L(\text{overall}) = f_L / \sqrt{2^{(1/N)} - 1}$ . This formula shows that as  $N$  increases, the denominator decreases, causing  $f_L(\text{overall})$  to increase (shift higher), thus reducing the low-frequency response.

- **Overall Upper Cutoff Frequency ( $f_H(\text{overall})$ ):**

- The overall upper cutoff frequency of a cascaded amplifier is primarily determined by the *lowest* of the individual upper cutoff frequencies of the stages. If one stage has an  $f_H$  of 1 MHz and another has 500 kHz, the 500 kHz cutoff will dominate the high-frequency performance of the entire amplifier. Signals above 500 kHz will be significantly attenuated by that particular stage, limiting the overall high-frequency response.
- For  $N$  identical cascaded stages, the overall upper cutoff frequency is slightly lower than that of a single stage. The precise calculation for  $N$  identical stages where  $f_H$  is due to a single dominant pole is:  $f_H(\text{overall}) = f_H * \sqrt{2^{(1/N)} - 1}$ . This formula shows that as  $N$  increases, the multiplier decreases, causing  $f_H(\text{overall})$  to decrease (shift lower), thus reducing the high-frequency response.

- **Overall Mid-Band Gain:**

- The overall mid-band voltage gain of a cascaded amplifier is the product of the individual mid-band voltage gains of each stage (assuming no loading effects are already factored into individual gains).
- Formula (linear scale):  $A_{v\_overall} = A_{v1} * A_{v2} * \dots * A_{vN}$
- Formula (decibel scale):  $A_{v\_overall\_dB} = A_{v1\_dB} + A_{v2\_dB} + \dots + A_{vN\_dB}$
- This implies that cascading stages increases the total gain, but at the cost of reduced bandwidth.

**Explanation of Bandwidth Reduction in Multistage Amplifiers:** Consider two identical amplifier stages, each with an upper cutoff frequency  $f_H$ . At  $f_H$ , the gain of a single stage drops by 3 dB. When a signal at  $f_H$  passes through the first stage, its amplitude is reduced by 3 dB. When this already attenuated signal then passes through the second stage at the same frequency, it is *again* reduced by 3 dB. So, for the two stages combined, the signal at  $f_H$  experiences a total attenuation of 6 dB. For the overall bandwidth of the cascaded amplifier to be defined at the -3 dB point (relative to the overall mid-band gain), this -3 dB point must occur at a *lower* frequency than the individual  $f_H$  of each stage, where the combined attenuation is exactly 3 dB. A similar logic applies to the lower cutoff frequency, where the combined gain drop-off occurs at a higher frequency.

**Numerical Example 4.3.1:** A three-stage amplifier system is constructed from three distinct amplifier stages.

- Stage 1:  $f_{L1} = 20 \text{ Hz}$ ,  $f_{H1} = 1 \text{ MHz}$ ,  $\text{Gain}_1 = 10$  (20 dB)
  - Stage 2:  $f_{L2} = 50 \text{ Hz}$ ,  $f_{H2} = 800 \text{ kHz}$ ,  $\text{Gain}_2 = 15$  (23.5 dB)
  - Stage 3:  $f_{L3} = 10 \text{ Hz}$ ,  $f_{H3} = 1.2 \text{ MHz}$ ,  $\text{Gain}_3 = 8$  (18.1 dB)
  - **Problem:** Estimate the overall lower and upper cutoff frequencies and calculate the overall mid-band voltage gain (in both linear and dB).
  - **Part A: Estimate Overall Lower Cutoff Frequency ( $f_{L(\text{overall})}$ ).**
    - **Rule:** The highest individual  $f_L$  dominates.
    - **Comparison:**  $f_{L1} = 20 \text{ Hz}$ ,  $f_{L2} = 50 \text{ Hz}$ ,  $f_{L3} = 10 \text{ Hz}$ .
    - **Calculation:**  $f_{L(\text{overall})} \text{ approx} = \max(20 \text{ Hz}, 50 \text{ Hz}, 10 \text{ Hz}) = 50 \text{ Hz}$ .
    - **Result:** The estimated overall lower cutoff frequency is 50 Hz.
  - **Part B: Estimate Overall Upper Cutoff Frequency ( $f_{H(\text{overall})}$ ).**
    - **Rule:** The lowest individual  $f_H$  dominates.
    - **Comparison:**  $f_{H1} = 1 \text{ MHz}$ ,  $f_{H2} = 800 \text{ kHz}$ ,  $f_{H3} = 1.2 \text{ MHz}$ .
    - **Calculation:**  $f_{H(\text{overall})} \text{ approx} = \min(1,000 \text{ kHz}, 800 \text{ kHz}, 1,200 \text{ kHz}) = 800 \text{ kHz}$ .
    - **Result:** The estimated overall upper cutoff frequency is 800 kHz.
  - **Part C: Calculate Overall Mid-Band Voltage Gain.**
    - **Formula (linear):**  $A_{v\_overall} = A_{v1} * A_{v2} * A_{v3}$
    - **Calculation (linear):**  $A_{v\_overall} = 10 * 15 * 8 = 1200$ .
    - **Formula (dB):**  $A_{v\_overall\_dB} = A_{v1\_dB} + A_{v2\_dB} + A_{v3\_dB}$
    - **Calculation (dB):**  $A_{v\_overall\_dB} = 20 \text{ dB} + 23.5 \text{ dB} + 18.1 \text{ dB} = 61.6 \text{ dB}$ .
    - **Result:** The overall mid-band voltage gain is 1200 (or 61.6 dB).
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## 4.4 Cascode Amplifier

The cascode amplifier is a specialized two-stage amplifier configuration widely recognized for its superior high-frequency performance, high output impedance, and improved reverse isolation. It achieves these advantages by combining the characteristics of two different basic amplifier configurations in series.

### 4.4.1 Structure and Operation:

A typical BJT cascode amplifier consists of two transistors (let's use Q1 and Q2) arranged as follows:

1. **Input Stage (Q1):** This is a common-emitter (CE) configuration. The input signal ( $V_{in}$ ) is applied to its base. Its emitter is connected to ground (or a common reference through a bias resistor). The collector of Q1 is directly connected to the emitter of Q2.
2. **Output Stage (Q2):** This is a common-base (CB) configuration. Its emitter is connected to the collector of Q1. Its base is held at a fixed DC voltage (usually set by a voltage divider). The output ( $V_{out}$ ) is taken from the collector of Q2. The collector of Q2 is connected to the positive power supply via a load resistor.

### Detailed Operation:

- **Q1 (Common-Emitter Stage):** The input signal applied to the base of Q1 causes variations in its collector current. In a standard CE amplifier, these collector current variations would cause significant voltage swings at the collector, which, via the Miller effect, would feed back to the input through the collector-base capacitance ( $C_{\mu 1}$ ) and limit the high-frequency response.
- **Q2 (Common-Base Stage):** This is where the magic of the cascode lies. The emitter of Q2 is connected directly to the collector of Q1. A key characteristic of a common-base configuration is its **very low input impedance at the emitter**. This low input impedance effectively "fixes" the voltage at the collector of Q1 to be almost constant, regardless of the input signal variations at Q1's base.
- **Reduced Miller Effect:** Because the voltage at Q1's collector remains relatively constant (minimal voltage swing), the voltage swing across Q1's collector-base capacitance ( $C_{\mu 1}$ ) is drastically reduced. This, in turn, virtually eliminates the Miller multiplication of  $C_{\mu 1}$  at the input of Q1. The effective input capacitance of Q1 is thus dominated by its  $C_{\pi 1}$ , which is generally much smaller than the Miller-amplified  $C_{\mu 1}$  in a standard CE stage. This is the primary reason for the cascode's superior high-frequency performance.
- **Current Transfer:** Q1 acts as a transconductance stage, converting the input voltage signal into a current signal ( $I_{c1}$ ). This current signal is then fed directly into the emitter of Q2.
- **Output from Q2:** Q2, being a common-base stage, essentially functions as a current buffer that transfers the current from its emitter ( $I_{c1}$ ) to its collector ( $I_{c2} \approx I_{c1}$ ) with little voltage gain but with a high output impedance. The output voltage is then generated across the load resistor connected to Q2's collector.
- **Overall Gain:** The overall voltage gain of the cascode amplifier is approximately the product of the transconductance of Q1 ( $g_{m1}$ ) and the output resistance of Q2 ( $r_{out\_Q2}$ ). Since the common-base stage provides very little reverse voltage gain, it effectively isolates the input from the output, further improving stability at high frequencies.

#### 4.4.2 Advantages of the Cascode Amplifier:

1. **Improved Bandwidth / Higher Upper Cutoff Frequency:** This is the most significant advantage. By suppressing the Miller effect in the input common-emitter (or common-source) transistor, the dominant pole that limits high-frequency response is shifted to a much higher frequency. This results in a wider bandwidth compared to a single common-emitter/common-source stage with comparable gain.
2. **Higher Output Impedance:** The output of the cascode amplifier is taken from the collector of the common-base stage (Q2). A common-base configuration intrinsically offers a very high output impedance, making the cascode amplifier suitable for driving high-impedance loads or for use as a high-impedance current source.
3. **Higher Reverse Isolation (Reduced Feedback):** The common-base stage acts as an effective shield or buffer between the input and output. It prevents unwanted signal feedback from the output to the input through parasitic capacitances (like the collector-base capacitance of Q2), which can cause instability or oscillation at high frequencies.
4. **Higher Voltage Gain (in some implementations):** While the common-base stage itself has a voltage gain close to unity (or slightly less), its very high output



impedance allows for a larger voltage gain when driving a resistive load, contributing to overall high gain without sacrificing bandwidth due to Miller effect in the input stage.

5. **Reduced Early Effect:** Because the voltage at the collector of Q1 is held relatively constant, the Early effect in Q1 is also minimized, leading to a higher effective output resistance for Q1, which can contribute to overall gain.

#### Disadvantages:

1. **Higher Supply Voltage Requirement:** The stacked nature of the two transistors means that a higher DC supply voltage is generally required to ensure both transistors operate correctly in their active regions (i.e., sufficient  $V_{ce}$  for both Q1 and Q2).
2. **Increased Complexity and Component Count:** A cascode amplifier uses two transistors and requires more complex biasing circuitry than a simple single-stage amplifier.
3. **Limited Output Voltage Swing:** Due to the stacked configuration, the maximum possible output voltage swing can be limited, especially in lower supply voltage applications.

**Applications:** Cascode amplifiers are widely used in critical high-frequency applications where wide bandwidth, high input impedance, and high reverse isolation are crucial. Examples include RF amplifiers, the input stages of wideband oscilloscopes, professional audio equipment, and operational amplifier input stages.

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## 4.5 Introduction to Power Amplifiers

Up to this point, our discussions have largely centered on voltage amplifiers (also known as small-signal amplifiers), whose primary function is to increase the voltage amplitude of an input signal while maintaining its waveform fidelity. These amplifiers are typically designed to operate within the linear region of the transistor's characteristics, handling relatively small signal swings and usually driving high-impedance loads. However, many real-world applications require more than just voltage amplification; they demand **power amplification**, which involves delivering significant power to a load.

### 4.5.1 Need for Power Amplification:

The fundamental purpose of a power amplifier (sometimes called a large-signal amplifier) is to increase the power level of an input signal to a magnitude sufficient to drive a specific load. This implies not only a large voltage swing but also the ability to supply substantial current. Unlike voltage amplifiers, power amplifiers are optimized for efficiency and maximum power delivery, often operating closer to the saturation or cutoff regions of the transistor characteristic curves.

Consider the distinction: a voltage amplifier might take a weak signal from a microphone (a few millivolts) and amplify it to several volts, but with only microamperes of current. This amplified voltage is sufficient to drive another amplifier stage, which has a high input

impedance. However, to actually produce sound from a loudspeaker, several volts and amperes of current are typically required. This is where the power amplifier comes in.

Examples of applications necessitating power amplification and the driving of low-impedance loads include:

- **Loudspeakers:** These are perhaps the most common example. A loudspeaker typically has an impedance ranging from 4 to 16 Ohms. To produce audible sound, it requires power, which translates to significant voltage and current swings. For instance, to deliver 100 Watts to an 8 Ohm loudspeaker requires an RMS voltage of 28.3 V and an RMS current of 3.54 A.
- **Antennas:** In wireless communication systems (e.g., radio transmitters, cellular base stations), power amplifiers boost the radio frequency (RF) signal to a high power level before feeding it to an antenna for efficient transmission over long distances.
- **Electric Motors and Actuators:** In control systems and robotics, electric motors, solenoids, and other actuators require considerable current and voltage to generate torque or force. Power amplifiers (often specialized as motor drivers) supply this energy.
- **Heating Elements and Solenoids:** Applications involving direct conversion of electrical energy to heat or magnetic force often rely on power amplifiers or high-current drivers.
- **Sonar and Ultrasound Transducers:** These devices convert electrical energy into acoustic waves (and vice-versa) and require power amplifiers to generate strong sound pulses for imaging or ranging.

Power amplifiers are characterized not just by their voltage gain, but critically by their maximum power output, their power efficiency (how much DC power is converted to useful AC power), and their linearity (how faithfully the amplified waveform reproduces the input signal, especially at large amplitudes). Thermal management (dissipating the heat generated due to inefficiencies) is also a major design consideration.

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## 4.6 Classes of Operation in Power Amplifiers

Power amplifiers are categorized into different "classes" of operation. These classes define the portion of the input signal cycle during which the amplifying transistor (or active device) conducts current. Each class represents a specific trade-off between power efficiency (how much DC power is converted to useful AC power) and linearity (how faithfully the output signal reproduces the input signal, i.e., how little distortion is introduced).

### 4.6.1 Class A Amplifier:

- **Characteristics:**
  - **Conduction Angle:** The active device (transistor) conducts current for the **entire 360 degrees** (100%) of the input signal's cycle. This means the transistor is always "on" and operating in its active (linear) region, regardless of the instantaneous input signal amplitude.

- **Biasing:** The quiescent operating point (Q-point) of the transistor is set near the center of the DC load line. This ensures that the transistor never enters cutoff (turns off) or saturation (fully turns on/short-circuit behavior) during the entire swing of the input signal, thereby keeping it in the linear operating region.
- **Output Waveform:** Produces an output waveform that is a very accurate and faithful (highly linear) reproduction of the input signal. This results in very low harmonic and intermodulation distortion.
- **Quiescent Power Dissipation:** A significant disadvantage is that the Class A amplifier constantly draws current from the power supply, even when there is no input signal applied. This continuous current flow results in quiescent power dissipation, which is wasted as heat.
- **Efficiency:**
  - The maximum theoretical efficiency of a Class A amplifier is very low.
    - For a **resistively coupled load (e.g., common emitter with a collector resistor)**, the maximum theoretical efficiency is **25%**. This is because, even with no signal, the transistor and the load resistor are dissipating power. When a signal is applied, the maximum AC power that can be delivered to the load is limited, and a significant portion of the DC input power is still dissipated as heat by the transistor.
    - For a **transformer-coupled load**, the maximum theoretical efficiency can reach **50%**. This improvement comes because the transformer isolates the DC quiescent current from the load and allows for better impedance matching.
  - This low efficiency means that a large portion of the DC power consumed is converted into heat rather than useful output power, requiring substantial heat sinks for even moderate power outputs.
- **Applications:** Primarily used in low-power audio preamplifiers, headphone amplifiers, driver stages for other power amplifier classes, or in applications where signal fidelity and linearity are of utmost importance and power consumption is a secondary concern.

#### 4.6.2 Class B Amplifier:

- **Characteristics:**
  - **Conduction Angle:** Each active device (transistor) conducts current for **only 180 degrees (50%)** of the input signal cycle.
  - **Biasing:** Typically, the transistors are biased at cutoff (or very close to it). This means that ideally, no current flows when there is no input signal.
  - **Configuration:** Class B amplifiers are almost exclusively implemented using a **push-pull configuration**. This involves two complementary transistors (e.g., NPN and PNP BJT, or N-channel and P-channel MOSFETs). One transistor amplifies the positive half-cycle of the input signal, and the other amplifies the negative half-cycle. The two halves are then combined at the output.
- **Crossover Distortion:**
  - This is the major drawback of Class B operation. Because each transistor requires a small turn-on voltage (e.g., 0.7 V for a silicon BJT V<sub>BE</sub>), there's a

brief period around the zero-crossing of the input signal where neither transistor is sufficiently biased to conduct. This creates a "dead band" or discontinuity in the output waveform, resulting in noticeable distortion, especially at low signal levels. This distortion is called **crossover distortion**.

- **Efficiency:**
  - The maximum theoretical efficiency of a Class B amplifier is **78.5%**. This significantly higher efficiency compared to Class A is because power is drawn from the supply only when a signal is present and only for half of the cycle, reducing quiescent power dissipation.
- **Applications:** Historically used in audio power amplifiers, but less common now due to crossover distortion. More often found in applications where efficiency is critical and some distortion is acceptable, or where the distortion can be mitigated by other means.

#### 4.6.3 Class AB Amplifier:

- **Characteristics:**
  - **Conduction Angle:** Each active device conducts current for **slightly more than 180 degrees** (e.g., typically 185 to 200 degrees) of the input signal cycle.
  - **Biasing:** This class is a compromise between Class A and Class B. A small, carefully chosen quiescent bias current is applied to each transistor (a small "trickle" current). This ensures that both transistors are slightly "on" simultaneously around the zero-crossing point of the input signal.
- **Mitigating Crossover Distortion:** The small overlap in conduction near the zero-crossing effectively eliminates the "dead band" that causes crossover distortion in Class B. This results in a much smoother transition between the positive and negative halves of the amplified signal, significantly improving linearity and fidelity.
- **Efficiency:**
  - The maximum theoretical efficiency of a Class AB amplifier is slightly less than Class B, typically ranging from **60% to 75%**. The exact efficiency depends on the amount of quiescent bias current chosen. The small quiescent power dissipation reduces efficiency compared to Class B, but it is still vastly more efficient than Class A.
- **Applications:** Class AB is the **most widely used class for high-fidelity audio power amplifiers** today due to its excellent balance of high linearity (low distortion) and good efficiency. It provides near Class A sound quality with Class B efficiency.

#### 4.6.4 Class C Amplifier:

- **Characteristics:**
  - **Conduction Angle:** The active device conducts current for **significantly less than 180 degrees** of the input signal cycle (typically 90 to 150 degrees).
  - **Biasing:** The transistor is biased deeply into cutoff. It only conducts for a brief pulse when the input signal's amplitude is large enough to push it above the cutoff threshold.

- **Output Waveform:** Produces a highly distorted output waveform, as only a small fraction of the input signal is amplified into current pulses. It does not reproduce the input waveform faithfully.
- **Efficiency:**
  - Class C amplifiers offer the **highest theoretical efficiency, potentially approaching 100%**. This is because the transistor is in cutoff for most of the cycle, meaning minimal power is dissipated by the transistor itself. Power is only consumed during the short pulses of conduction.
- **Applications (Tuned Amplifiers):**
  - Class C amplifiers are **not suitable for audio frequency (AF) amplification** due to their extreme non-linearity and high distortion.
  - They are primarily used in **radio frequency (RF) tuned amplifiers**, particularly in transmitters. Here, the highly distorted current pulses from the Class C amplifier are fed into a parallel resonant (LC) tank circuit. The tank circuit acts as a filter, "ringing" at its resonant frequency and effectively reconstructing a clean sinusoidal output at the desired fundamental frequency, while filtering out all the generated harmonics.
  - This makes them highly efficient for generating single-frequency (or narrow-band) RF signals.

**4.6.5 Other Classes (Brief Overview):** Beyond the fundamental Class A, B, AB, and C, several other amplifier classes exist, often representing variations or more advanced topologies designed to push the boundaries of efficiency or linearity for specific applications.

- **Class D:** These are **switching amplifiers**. Instead of operating transistors in their linear region, Class D amplifiers convert the analog input signal into a series of digital pulses, typically using Pulse-Width Modulation (PWM). The transistors then operate in either a fully ON (saturated) or fully OFF (cutoff) state. When a transistor is fully ON, its voltage drop is minimal, and when fully OFF, its current is minimal, so power dissipation ( $I \cdot V$ ) is extremely low in both states. An output low-pass filter reconstructs the amplified analog signal from the PWM pulses.
  - **Efficiency:** Offers extremely high theoretical efficiency (often quoted as close to 100%, practically 90-95%) due to minimal power loss in the switching transistors.
  - **Applications:** Widely used in modern digital audio systems (e.g., car stereos, home theater systems, portable devices), compact power supplies, and motor control.
- **Class G and Class H:** These are variations designed to improve the efficiency of linear (Class AB) amplifiers by minimizing the voltage drop across the output transistors. They achieve this by using **multiple power supply rails** (Class G) or **dynamically varying the power supply voltage** (Class H) to the output stage. The supply voltage tracks the signal envelope, providing only the necessary voltage to the output transistors. This reduces the average voltage drop across the transistors, leading to lower power dissipation and higher efficiency, especially at lower signal levels.
  - **Efficiency:** Significantly better than Class AB, while maintaining good linearity.
  - **Applications:** High-power audio amplifiers, public address systems.

- **Class S, Class T, etc.:** These are often specialized or proprietary amplifier classes, sometimes variations of switching amplifiers (like Class D) or hybrid designs, aiming for specific performance optimizations (e.g., further efficiency gains, specific sonic characteristics).

## 4.7 Power Efficiency and Linearity Issues in Power Amplifiers

The design and selection of power amplifiers inherently involve navigating a fundamental trade-off between **power efficiency** and **linearity (signal fidelity)**. Optimizing for one often comes at the expense of the other. Understanding these two critical performance metrics and their interrelationship is essential for any power amplifier application.

### 4.7.1 Power Efficiency ( $\eta$ ):

- **Definition:** Power efficiency is a dimensionless ratio that quantifies how effectively an amplifier converts the DC power supplied by its power source into useful AC signal power delivered to the load. It is a measure of the power conversion capability of the amplifier.
- **Formula:**  $\eta = (P_{\text{out(AC)}} / P_{\text{in(DC)}}) * 100\%$  Where:
  - $P_{\text{out(AC)}}$ : The average AC power delivered to the load. This is the useful power that drives the speaker, antenna, or motor. For a sinusoidal output,  $P_{\text{out(AC)}} = (V_{\text{rms\_out}})^2 / R_{\text{load}} = (I_{\text{rms\_out}})^2 * R_{\text{load}} = (V_{\text{peak\_out}})^2 / (2 * R_{\text{load}})$ .
  - $P_{\text{in(DC)}}$ : The average DC power drawn from the power supply. This is the total power consumed by the amplifier, including power dissipated as heat and power delivered to the load. For a single supply voltage  $V_{\text{cc}}$  and average supply current  $I_{\text{cc(avg)}}$ ,  $P_{\text{in(DC)}} = V_{\text{cc}} * I_{\text{cc(avg)}}$ .
- **Importance:** High efficiency is critically important for several reasons:
  - **Battery Life:** In portable, battery-powered devices (e.g., smartphones, portable audio players), high amplifier efficiency directly translates to longer battery life.
  - **Energy Consumption and Cost:** For high-power applications (e.g., public address systems, industrial drives, radio transmitters), inefficient amplifiers waste a large amount of electrical energy, leading to higher operating costs.
  - **Thermal Management:** The power that is not delivered to the load is dissipated as heat within the amplifier's components, primarily the transistors. Inefficient amplifiers generate more heat, requiring larger, more expensive, and often noisy cooling solutions (e.g., heat sinks, fans). Excessive heat can also reduce component lifespan.
  - **Size and Weight:** Smaller heat sinks and power supplies due to higher efficiency contribute to more compact and lighter amplifier designs.
- **Factors Affecting Efficiency:**
  - **Class of Operation:** As discussed in section 4.6, the class of operation is the primary determinant of theoretical efficiency. Class C and D offer the highest efficiencies, while Class A offers the lowest. This is directly related to the

conduction angle and how much time the active device spends in linear operation versus cutoff or saturation.

- **Quiescent Power Dissipation:** Power drawn from the supply even when no input signal is present (or when the signal is zero). Class A has high quiescent power, Class AB has a small amount, and Class B ideally has zero. This quiescent power is entirely wasted as heat and directly reduces overall efficiency.
- **Voltage Drop Across Transistors:** When a transistor is conducting and is in its active region, there is a voltage drop across its collector-emitter (BJT) or drain-source (FET) terminals. The product of this voltage drop and the current flowing through the transistor ( $V_{CE} \cdot I_C$  or  $V_{DS} \cdot I_D$ ) represents the power dissipated by the transistor itself. Reducing this voltage drop (e.g., by pushing the transistor closer to saturation, or using dynamic supply rails as in Class G/H) can improve efficiency.
- **Output Stage Saturation:** Driving transistors hard into saturation (like in Class D switching amplifiers) minimizes the voltage drop across them, leading to very low power dissipation and high efficiency. However, this is a non-linear operation.

**Numerical Example 4.7.1:** A Class AB power amplifier delivers a maximum average AC output power of 75 W to a loudspeaker. When delivering this power, it draws an average DC current of 5 A from a 20 V DC power supply.

- **Problem:** Calculate the power efficiency of the amplifier at maximum output.
- **Given:**  $P_{out}(AC) = 75 \text{ W}$ ,  $V_{cc} = 20 \text{ V}$ ,  $I_{cc}(avg) = 5 \text{ A}$ .
- **Step 1: Calculate the total DC input power ( $P_{in}(DC)$ ).**
  - **Formula:**  $P_{in}(DC) = V_{cc} \cdot I_{cc}(avg)$
  - **Calculation:**  $P_{in}(DC) = 20 \text{ V} \cdot 5 \text{ A} = 100 \text{ W}$ .
- **Step 2: Calculate the power efficiency ( $\eta$ ).**
  - **Formula:**  $\eta = (P_{out}(AC) / P_{in}(DC)) \cdot 100\%$
  - **Calculation:**  $\eta = (75 \text{ W} / 100 \text{ W}) \cdot 100\% = 0.75 \cdot 100\% = 75\%$ .
- **Result:** The efficiency of the amplifier at maximum output is 75%. This is a typical value for a well-designed Class AB amplifier operating near its maximum output.

#### 4.7.2 Linearity Issues:

- **Definition:** Linearity (or fidelity) refers to how accurately an amplifier reproduces the input signal at its output. A perfectly linear amplifier would produce an output waveform that is an exact scaled (amplified) replica of the input, without any alteration to its shape or introduction of new frequency components. Deviation from this ideal behavior is known as **distortion**.
- **Types of Distortion:**
  - **Harmonic Distortion (THD - Total Harmonic Distortion):** This occurs when the amplifier generates new frequency components that are integer multiples (harmonics) of the fundamental input signal frequency. For example, if the input is a pure 1 kHz sine wave, a non-linear amplifier might produce output components at 2 kHz, 3 kHz, 4 kHz, etc. These harmonics are undesirable

and degrade the signal quality. THD is usually quantified as the ratio of the RMS sum of the harmonic voltages to the RMS voltage of the fundamental.

- **Intermodulation Distortion (IMD):** This type of distortion occurs when multiple frequencies are present simultaneously in the input signal. The non-linear transfer characteristic of the amplifier causes these frequencies to interact, producing new frequency components that are sums and differences of the input frequencies and their harmonics (e.g., if inputs are  $f_1$  and  $f_2$ , IMD can produce  $f_1+f_2$ ,  $f_1-f_2$ ,  $2f_1+f_2$ , etc.). IMD is often considered more audibly irritating than harmonic distortion.
- **Crossover Distortion:** As discussed for Class B amplifiers, this specific type of distortion results from the "dead band" around the zero-crossing where the active devices momentarily turn off, leading to discontinuities in the output waveform.
- **Clipping Distortion:** Occurs when the input signal amplitude is too large, causing the output signal to exceed the amplifier's power supply rails. This forces the output waveform to flatten at the peaks, introducing severe non-linearity and producing many harmonics.
- **Causes of Non-Linearity:**
  - **Transistor Transfer Characteristics:** The relationship between input (e.g.,  $V_{BE}$  for BJT,  $V_{GS}$  for FET) and output current ( $I_C$  for BJT,  $I_D$  for FET) for transistors is inherently non-linear, especially over large signal swings. For example, the exponential relationship of  $I_C$  vs  $V_{BE}$  in a BJT contributes to harmonic distortion.
  - **Biasing:** Incorrect or inadequate biasing can push the transistor's operation into highly non-linear regions (e.g., too close to cutoff or saturation), particularly for large signals.
  - **Load Variations:** Reactive loads (e.g., loudspeakers with varying impedance across frequency) can cause non-linear loading effects.
  - **Power Supply Imperfections:** Non-ideal power supplies (e.g., voltage droop under heavy load) can lead to distortion.
- **Importance:** High linearity is paramount in applications where signal integrity is critical.
  - **High-Fidelity Audio:** In home audio systems, low distortion is essential for accurate sound reproduction that is pleasing to the listener.
  - **Precision Instrumentation:** Amplifiers used in measurement equipment must be highly linear to ensure accurate readings.
  - **Communication Systems:** In radio transmitters and receivers, distortion can cause interference, reduce signal quality, and lead to errors in data transmission.
  - **Medical Electronics:** Distortion in signals can lead to misinterpretation of medical data (e.g., in ECG or EEG systems).

**Trade-off between Efficiency and Linearity:** This is the central challenge in power amplifier design.

- **Class A amplifiers** achieve the highest linearity because they operate purely in the linear region of the transistor's characteristics. However, this comes at the cost of very low efficiency due to continuous power dissipation.



- **Class B amplifiers** offer significantly higher efficiency by allowing the transistors to turn off for half of the cycle, but this introduces inherent crossover distortion, compromising linearity.
- **Class AB amplifiers** strike a balance. By providing a small quiescent bias, they overcome crossover distortion, achieving high linearity comparable to Class A, while maintaining reasonably high efficiency, albeit slightly less than pure Class B. This compromise makes Class AB the most popular choice for general-purpose linear audio amplification.
- **Class C and D amplifiers** achieve the highest efficiencies by operating the transistors in a highly non-linear fashion (cutoff/saturation). They are inherently non-linear and are therefore only suitable for applications where the resulting distortion can be managed (e.g., by using resonant circuits in Class C RF amplifiers) or is acceptable.

Modern power amplifier designs often employ negative feedback techniques to improve linearity across all classes. By feeding a portion of the output signal back to the input in a phase-opposing manner, feedback can significantly reduce distortion, but it can also introduce stability issues or reduce the overall gain. The choice of power amplifier class is always a careful consideration of the specific application's requirements for power output, efficiency, cost, heat dissipation, and the acceptable level of signal distortion.